

Subsurface Resistivity Measurements Using Square Waveforms

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Abstract— This work analyzes the effect of inductive and capacitive coupling between the injecting circuit and the detecting circuit in resistive field surveys. Theoretical and experimental results demonstrate that if a square waveform is injected into the soil, and synchronous sampling is used to sample at the flat zone of the detected voltage, then the effect of the interference is greatly reduced. Furthermore, square waveforms are easier to generate than sinusoidal waveforms, so they offer a new approach to subsurface resistivity measurements.

Index Terms— Conductivity measurement, data acquisition, electromagnetic coupling, sampling methods, soil measurements, synchronous detection.

I. INTRODUCTION

THE detection of buried structures from the surface without drilling into the soil is of interest in archaeology and in other applications such as detection of water and contaminants from leaking underground pipes. Ground penetrating radar and time domain reflectometry are techniques that can be applied in these cases. Our work is concerned with geoelectrical prospecting methods, which consist of injecting a current to the soil with a pair of electrodes and detecting the drop in voltage with another pair of electrodes.

The parameter that provides information on the buried structure (the anomaly) is the apparent resistivity ρ_a , which is given by $\rho_a = k\Delta V/I$ [1]. In this expression ΔV is the detected voltage, I is the injected current, and k is a geometric factor, which depends on the configuration of the electrode array. It is important to recognize that any possible error in the measured voltage will affect ρ_a and can hinder the detection of the anomaly. Some error sources are the position of the electrodes, telluric noise (which has its main influence in dc measurements) and electromagnetic coupling between the injecting and detecting circuits in ac measurements.

In electrical surveys, electromagnetic induction between current and voltage cables must be avoided. This is easily achieved by using dipole–dipole arrays [2], but it is interesting to devise a measurement method able to reduce this interference independently of electrode configuration.

II. PROBLEM STATEMENT

In electrical impedance measurement, we inject a current (frequency f_s) and detect a drop in voltage whose amplitude

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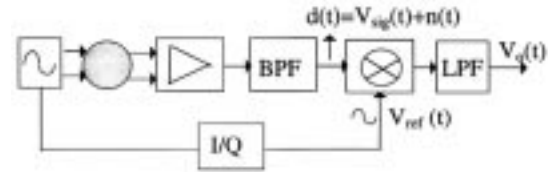


Fig. 1. Impedance measurement by homodyne detection.

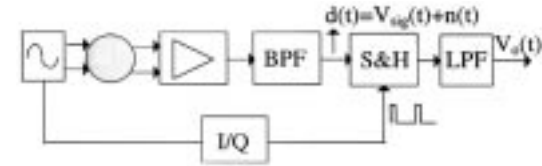


Fig. 2. Impedance measurement by synchronous sampling.

is modulated by the impedance sensed. Modulation produces an upward translation of the information spectrum. Demodulation, therefore, implies a downward frequency translation in order to recover the information from the modulated wave.

A common demodulation technique providing a good signal-to-noise ratio is homodyne detection. This method can be applied to impedance measurement as shown in Fig. 1.

If we assume that the signal detected is $V_{sig}(t) = \sqrt{2}V_s \cos(2\pi f_s t + \phi_s)$ and the reference signal is $V_{ref}(t) = \sqrt{2}V_r \cos 2\pi f_r t$, then the demodulated signal at the output of the low pass filter, when $f_s = f_r$, is $V_o(t) = V_s V_r \cos \phi_s$. In a resistive medium, $\phi_s = 0$, then $V_o(t) = V_s V_r$.

If there is an interfering signal $n(t) = \sqrt{2}V_i \cos(2\pi f_i t + \phi_i)$, having the same frequency than the carrier, typically capacitively or inductively coupled, then the demodulated signal in a resistive medium is $V_o(t) = V_r(V_s + V_i \cos \phi_i)$. If the phase angle ϕ_i of the interference is 90° then the signal will be recovered without any error. But when $\phi_i \neq 90^\circ$ there will be a measurement error. In principle, the larger the frequency of the injected current, the larger the interference will be, because both inductive and capacitive interference increase with frequency. In extreme cases the detector can even saturate because of the interference.

Impedance measurement by synchronous sampling (Fig. 2) is another phase-sensitive amplitude demodulation technique [3]. If $d(t)$ is sampled at $t = nT$, n integer, T being the period of the signal, that is, if the signal $V_{sig}(t)$ is sampled at its maximal value, then the detected signal will be $V_o(t) = \sqrt{2}V_s(1 + (V_i/V_s) \cos \phi_i)$. It can be seen that the measurement error also depends on the phase angle ϕ_i of

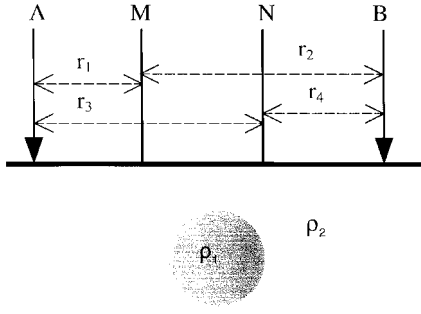


Fig. 3. Linear array of current and potential electrodes.

the interference. But here this drawback can be avoided by injecting a square waveform to the soil instead of a sinusoidal waveform [4]. In this case the detected signal is sampled at a zero-slope point, thus avoiding the stray current coupling from the injecting circuit to the measuring circuit (“transformer effect”). Therefore, this allows us to measure at a frequency high enough in order for electrode impedance to be relatively low.

Electromagnetic interference is particularly troublesome in subsurface resistivity measurements because of the physical dimensions and arrangement of circuits. Fig. 3 shows the principle of this technique [1]. A current of strength I is injected by electrodes A and B . The potential difference between points M and N in a homogeneous medium is

$$\Delta V = V_M - V_N = \frac{I\rho}{2\pi} \left[\left(\frac{1}{r_1} - \frac{1}{r_2} \right) - \left(\frac{1}{r_3} - \frac{1}{r_4} \right) \right] \quad (1)$$

where ρ is the soil resistivity, and r_1 to r_4 are distances between the electrodes. The apparent resistivity ρ_a is obtained by solving (1) for ρ :

$$\rho_a = k \frac{\Delta V}{I} \quad (2)$$

where k is the geometric factor.

The apparent resistivity provides information about the presence of an anomaly. The potential difference ΔV can be affected by the electromagnetic coupling from the injecting circuit to the detecting circuit, which will alter the value of ρ_a .

There are two main coupling mechanisms: capacitive coupling and inductive coupling [5]. Capacitive coupling arises from the parasitic capacitances between injecting and detecting wires. Inductive coupling appears because the injecting and detecting circuits behave like the primary and secondary winding of an ordinary transformer.

If dc currents are used, then there is neither capacitive nor inductive coupling. But dc currents pose another important problem: polarization potentials generated at the contact between a metallic conductor (the electrode) and an electrolytic conductor (the moist ground). These polarization potentials are dc voltages that mask the measured voltage difference.

Interference by capacitive and inductive coupling results only when there is a change in the voltage or current in the injecting circuit. For a sinusoidal signal, this means the entire waveform except at its maximum and minimum. For a square signal, however, interference will result only during transition

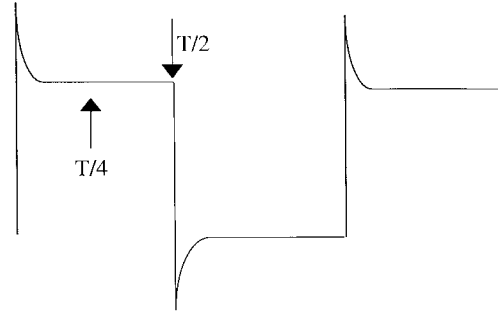
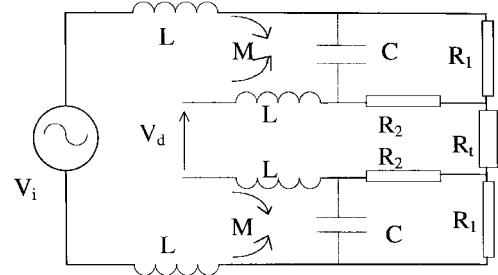
Fig. 4. Synchronous sampling with square waveforms. From $T/4$ to $T/2$ the interference is minimal.

Fig. 5. Inductive and capacitive coupling between injecting and detecting circuits.

times. Hence, by sampling at $T/4$ or a bit later the effect of the interference should disappear (Fig. 4).

III. THEORETICAL ANALYSIS

Fig. 5 shows the circuit model of a resistivity measurement. V_i and V_d are the injected and detected voltages, respectively, C is the capacitance between the injecting and detecting wires, L and M are the self and mutual inductance of cables, R_1 and R_2 are the contact resistances of the injecting and detecting electrodes, and R_t is the resistance between the electrodes M and N . For simplicity, we study capacitive and inductive interferences separately.

If no interference is present, the ideal detected voltage will be

$$V_{do} = \frac{R_t}{R_t + 2R_1} \cdot V_i. \quad (3)$$

We define the relative error in the measured voltage as

$$e_r = \frac{V_d - V_{do}}{V_{do}} \quad (4)$$

where V_d is the detected voltage contaminated by the interference. We consider an injected frequency of up to 10 kHz, R_t about 0.33 Ω , R_1 and R_2 in the range of 50 Ω to 5 k Ω , C ranging from 10–100 pF, and L and M between 10 and 100 μ H. Either for sinusoidal and square waveforms we sample V_d at $T/4 + nT$, where T is the period of the signal and n is an integer.

First of all, we consider the effect of stray capacitances. When we inject a square voltage whose peak value is V_{ip} the

detected voltage is

$$V_d \approx V_{ip} \left(\frac{R_t}{R_t + 2R_1} + \frac{4R_1}{R_t + 2R_1} e^{-t/CR_2} \right) \quad (5)$$

and the relative error for the assumed value range for the different parameters is

$$e_r \approx \frac{4R_1}{R_t} e^{-t/CR_2}. \quad (6)$$

If the generator injects a sinusoidal waveform, the relative error in the measured voltage for the in phase component is

$$e_r \approx \frac{2\omega^2 C^2 R_2^2 R_1}{R_t}. \quad (7)$$

It can be seen that the error greatly increases as R_2 and C become larger. For example, if the frequency is 10 kHz, $C = 10$ pF, and $R_1 = R_2 = 500 \Omega$ the relative error for sinusoidal signals is only 3×10^{-4} but when $C = 100$ pF and $R_1 = R_2 = 5 \text{ k}\Omega$ the relative error increases to 30. If square waveforms are used the error is negligible in both cases.

Secondly, we consider the effect of inductive coupling. The relative error for square signals in the same conditions as above is

$$e_r \approx \frac{4R_1}{R_t} e^{(-R_1/L)t} \quad (8)$$

and for sinusoidal signals is

$$e_r \approx \frac{2\omega^2 ML}{R_t R_1}. \quad (9)$$

It is interesting to point out that if R_1 is reduced the error increases in both cases. For sinusoidal signals the relative error varies from 48×10^{-4} when $L = M = 10 \mu\text{H}$ and $R_1 = R_2 = 500 \Omega$, to 4.68 when $L = M = 100 \mu\text{H}$ and $R_1 = R_2 = 50 \Omega$. This error is minimized to 22×10^{-4} when using square waveforms.

Electromagnetic coupling increases with frequency. To avoid the effect of interference in the measured voltage it is advisable therefore to inject a square waveform into the soil instead of a sine wave and to use a detector based on synchronous sampling.

IV. MATERIALS AND METHODS

In order to quantify interference in a field survey, we carried out some measurements in a typical soil of our campus. Fig. 6 shows the electrode arrangement. The injecting and detecting cables were arranged in a triangular geometry parallel to the ground surface. The transmitter and the receiver were placed in a vertex (TD) of this triangle, in front of the electrode array. The generator was an HP3245A and the detector was a fully differential synchronous demodulator, based on synchronous sampling [6]. The four electrodes were made from stainless steel, 20 cm in length and 1 cm in diameter. They were inserted several centimeters into the ground to ensure a good electrical contact. Connecting wires were 1 mm in diameter and had

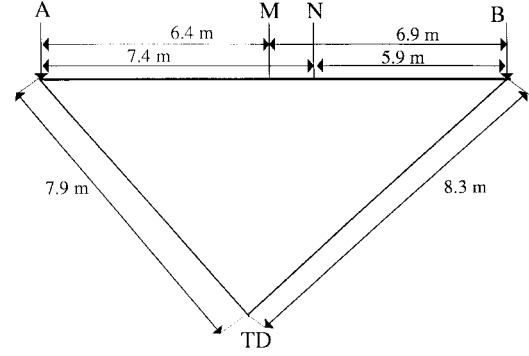


Fig. 6. Field survey: electrode array and wire arrangement.

plastic insulation, which avoided any possible direct leakage current to the ground.

The HP3245 generates the sinusoidal/square signal of 20 V_{pp} and the signal reference for the sampler. The sampling frequency was the same that the input signal ($f_r = f_s$) and its values were 100 Hz, 1 kHz, and 10 kHz. The detector had a gain of 100. The duty cycle for the sampling signal was 10%. The sampling instant was at $T/4 + nT$. The drop in voltage monitored the injected current across a 10Ω resistor in series with an injecting wire, by means of a portable oscilloscope (Tektronix THS-710E).

V. EXPERIMENTAL RESULTS

In order to quantify electromagnetic interference, we measured first the capacitive coupling, secondly the inductive interference and finally their combined effect in the differential voltage measured between electrodes M and N .

Capacitive interference increases in value if coupled to a high-impedance circuit. In order to demonstrate this effect, a variable resistor was connected to the end of two twisted cables. The detected potential increased with the resistor value. By placing a resistor of $5 \text{ k}\Omega$, at 10 kHz and by sampling at $T/4$ of the injected signal, 141 mV were detected at the output for a sinusoidal waveform and only 14 mV for a square waveform. Capacitive coupling increased with frequency and with cable length. Inductive interference was minimal because the detecting wires were twisted.

If the area of the detecting circuit is increased and its terminals short-circuited, capacitive interference is negligible (zero dependence) and inductive coupling predominates. At 10 kHz the detector output was 23 mV when using sinusoidal signals and only 2 mV by injecting square waveforms. Inductive coupling increased with the area of the detecting wires.

Table I shows the demodulated dc voltage V_o (once multiplied by 100) and the injected current (peak to peak) with sinusoidal and square waveforms at frequencies 1 and 10 kHz. The detector was connected to electrodes M and N . In this case there was, added to the ideal voltage, both capacitive and inductive interference and they increased with frequency. However, when using square waveforms the detected voltage was similar, which confirms our predictions that sampling in the flat zone of the square waveform, minimizes the effect of interference. When using sinusoidal waveforms, the voltage

TABLE I
DETECTED VOLTAGE WHEN INTERFERENCE COUPLING IS MAXIMUM

Injected wave	1 kHz	10 kHz
Sinusoidal Sampling instant= $T/4$	$V_o = 337 \text{ mV}$ $I = 19 \text{ mA}$	$V_o = 644 \text{ mV}$ $I = 20.6 \text{ mA}$
Square Sampling instant= $T/4$	$V_o = 351 \text{ mV}$ $I = 19 \text{ mA}$	$V_o = 370 \text{ mV}$ $I = 21 \text{ mA}$

TABLE II
DETECTED VOLTAGE WHEN DETECTING CABLES ARE TWISTED

Injected waveform	1 kHz	10 kHz
Sinusoidal Sampling instant= $T/4$	$V_o = 349 \text{ mV}$ $I = 19.4 \text{ mA}$	$V_o = 437 \text{ mV}$ $I = 20.8 \text{ mA}$
Square Sampling instant= $T/4$	$V_o = 356 \text{ mV}$ $I = 19.4 \text{ mA}$	$V_o = 371 \text{ mV}$ $I = 21.2 \text{ mA}$

detected changed by 91%. The small change observed in the square wave is principally due to the decreasing value in the contact resistance R_1 with frequency, which increases the injected current.

If the detecting cables were twisted, interference reduced because the distance between detecting wires and injecting wires increases and the area of the detecting circuit was much smaller. The results are shown in Table II.

VI. CONCLUSIONS

We have proposed a simple model to study the effects of capacitive and inductive interference in subsurface resistivity measurements. Theoretical analysis shows that injecting a square waveform into the soil instead of a sine wave and using synchronous sampling diminishes the effect of the interference.

Experimental results in a typical soil of our campus show that capacitive interference increases with the electrode

impedance, cable length and frequency. Then watering the electrodes reduces the capacitive interference. Inductive coupling increased with the area of the detecting wires and with the frequency. Twisting the detecting cables reduces the interference. However, in any case the interference effect is much smaller when using square waveforms. This is because synchronous sampling allows taking samples in the interval $T/4$ – $T/2$, when interference effect has disappeared.

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Ramón Pallás-Areny (F'98), for a photograph and biography, see this issue p. 29.

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